Title: Solving Quadratic Equations using the Quadratic Formula
Class: Math 100 or Math 107
Author: Jason Miner
Instructions to tutor: Read instructions and follow all steps for each problem exactly as given.
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Solving Quadratic Equations using the Quadratic Formula

Purpose:
This is intended to refresh your knowledge about solving quadratic equations using the quadratic formula.

Recall that a **quadratic equation** is an equation that can be written in the form $ax^2 + bx + c = 0$, with $a \neq 0$. For example, $3x^2 + 4x - 7 = 0$, $6 - x^2 = 2x$, and $x(x+6) = 14$ are all quadratic equations. Note that the second two equations would require a couple algebraic steps to be put into the form shown above.

You have seen in previous examples that a quadratic equation such as, $x^2 - 5x - 6 = 0$, can be solved by factoring and then using the zero-product property. But, what do we do if we have a quadratic that does not factor? There is a formula that will help us – in fact, using this formula, we will be able to solve every quadratic equation. This formula may be derived using a method called completing the square.

Quadratic Formula
The solutions of the quadratic equation $ax^2 + bx + c = 0$, where a, b, and c are real numbers with $a \neq 0$, are given by the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: Let's solve $x^2 - 5x - 6 = 0$ using two methods, factoring and the quadratic formula.

First by factoring:
$$\begin{aligned} & x^2 - 5x - 6 = 0 \\ \Rightarrow\ & (x-6)(x+1) = 0 \\ \Rightarrow\ & x - 6 = 0 \text{ or } x + 1 = 0 \\ \Rightarrow\ & x = 6 \text{ or } x = -1 \end{aligned}$$

Now let's try using the formula. Note that for this equation, $a = 1$, $b = -5$, and $c = -6$. All we have to do is substitute these into the appropriate spots in the formula and simplify.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-6)}}{2(1)} = \frac{5 \pm \sqrt{25+24}}{2} = \frac{5 \pm \sqrt{49}}{2} = \frac{5 \pm 7}{2}$$

Note that there are two solutions, indicated by the $\pm$ symbol. So $\frac{5+7}{2} = 6$ and $\frac{5-7}{2} = -1$.

In the previous example, we obtained the same solution that we got by factoring. So how do you know when you should factor or when you should use the formula? Well, if something factors easily, then that is the route to take. However if you do not see how something factors or it just doesn't factor, then you can use the formula. Let's look at another example.

Example: Solve $4x^2 + 6x + 1 = 0$.

Note that this quadratic does not appear to factor – this is why we need the quadratic formula.

For this problem, $a = 4$, $b = 6$, and $c = 1$.

So we have $x = \dfrac{-(6) \pm \sqrt{(6)^2 - 4(4)(1)}}{2(4)} = \dfrac{-6 \pm \sqrt{36-16}}{8} = \dfrac{-6 \pm \sqrt{20}}{8} = \dfrac{-6 \pm 2\sqrt{5}}{8} = \dfrac{-3 \pm \sqrt{5}}{4}$.

Our solutions are $x = \dfrac{-3+\sqrt{5}}{4}$ and $x = \dfrac{-3-\sqrt{5}}{4}$.

Example: Now it's your turn. Solve $x^2 - 2x - 6 = 0$.

First try to factor………….ok, that is enough of that!

For the formula, you first need to identify a, b, and c:

$a =$ ______ $b =$ ______ $c =$ ______

Now make your substitutions: $x = \dfrac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$

Simplify your answer. Did you get $x = \dfrac{2 \pm \sqrt{28}}{2}$? Simplify a bit more

Did you get the solutions $x = 1 + 2\sqrt{7}$ and $x = 1 - 2\sqrt{7}$? Good! Now try the following on your own.

1. Solve each of the following quadratic equations.

(a) $x^2 - 12x + 27 = 0$

(b) $3x^2 - 8x + 1 = 0$

(c) $t(3t+5) = 20$ (Be careful – first you must write this in the form $ax^2 + bx + c = 0$.)

(d) $y^2 - 3y + 5 = 0$

For this last one, did you end up with a negative number under the radical? What do you think this means? Well, if $b^2 - 4ac$, called the **discriminant**, is negative, then the quadratic equation has no real solutions. Later you will learn more on this and come up with a way to discuss nonreal solutions you obtained.

Check your answers – If you did not get these, consult a tutor for help.

1. (a) $x = 3, 9$ (b) $x = \dfrac{4 \pm \sqrt{13}}{3}$ (c) $t = \dfrac{-5 \pm \sqrt{265}}{6}$ (d) No real solutions